Homework 2:
Review Statistics & Random Numbers

The objective of Homework 2 is for students to review elements of probability and statistics, as well as to understand randomness in simulation. For submission, kindly submit your work for all questions in 1 PDF file and 1 Excel file.

Part 1: Review of Statistics

Exercise 1: Answer the following questions (provide only 1 sentence each answer):

a. What is a point estimate?
b. Describe the lower and upper endpoint of a confidence interval in terms of the point estimate.
c. Describe the center value of a confidence interval in terms of the point estimate.
d. How different is a 90% confidence interval, a 95% confidence interval, and a 97% confidence interval compared to each other?
e. What do the null/alternative hypotheses state?

Exercise 2: Two policies for replacing bearing are compared as below. Estimate difference in the mean cost per replacing bearing of each policy at 95% of confidence level (apply a hypothesis test).

Policy 1	Policy 2
13340	17556
12760	17160
13002	17808
13524	18012
13754	18200
13318	17936
13432	18450
14208	19398
13224	17612
13178	17956

Part 2: Generation of RNs

Exercise 1: Use the LCM to generate a sequence of five three-digit random integers and corresponding random numbers with:

a. $X_0 = 117, a = 43, c = 0, \text{and } m = 1000.$
b. $X_0 = 37, a = 7, c = 29, \text{and } m = 100.$

Exercise 2: Consider the multiplicative congruential generator ($c = 0$) under the following conditions:

a. $X_0 = 7, a = 11, m = 16.$
b. $X_0 = 8, a = 11, m = 16.$
c. $X_0 = 7, a = 7, m = 16.$
d. $X_0 = 8, a = 7, m = 16.$

Generate enough values in each case to complete a cycle. What implications can be drawn? Is the maximum period achieved?

Exercise 3: Develop your own combined linear congruential random-number generator.

Part 3: Test for RNs

Exercise 1: Frequency test (KS test)

Consider 15 random numbers that had been generated below:

0.44	0.65	0.13	0.92	0.72	0.24	0.56	0.91	0.43	0.58
0.26	0.39	0.34	0.17	0.05					

Using the KS test ($\alpha = 0.05$) to test the hypothesis that the numbers are uniformly distributed on the interval $[0,1]$.

Exercise 2: Test the following sequence of numbers for uniformity and independence using a procedure you learned:

0.594	0.928	0.515	0.055	0.507	0.351	0.262	0.797	0.788	0.442
0.097	0.798	0.227	0.127	0.474	0.825	0.007	0.182	0.929	0.852